Physics Letters B 780 (2018) 608–615

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Gapped fermionic spectrum from a domain wall in seven dimension

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ARTICLE INFO

Article history:
Received 17 January 2018
Accepted 13 March 2018
Available online 20 March 2018
Editor: M. Cvetič

ABSTRACT

We obtain a domain wall solution in maximally gauged seven dimensional supergravity, which interpolates between two AdS spaces and spontaneously breaks a $U(1)$ symmetry. We analyse frequency dependence of conductivity and find power law behaviour at low frequency. We consider certain fermions of supergravity in the background of this domain wall and compute holographic spectral function of the operators in the dual six dimensional theory. We find fermionic operators involving bosons with non-zero expectation value lead to gapped spectrum.

1. Introduction

Gauge-gravity duality [1–3] has been proved to be extremely useful in studying strongly coupled fermionic systems. One can consider a custom gravity theory in accordance with the symmetries of the dual field theory and use the duality to analyse various aspects of the latter. The gravity theory with charged AdS black hole in one higher dimension provides necessary computational techniques to study the fermionic systems. Indeed, it leads to fermionic excitations with scaling behaviour of non-Fermi liquids [4–6]. Subsequently, study of low energy behaviour of the system with constant charge and mass as well as that of relation between scaling exponent and dimension of dual operator in general dimensions appeared in [7]. It was found that turning on dipole coupling beyond a critical value gives rise to dynamically generated gap [8, 9] as found in Mott insulators. Charged Lifshitz black brane with dipole coupling was considered in [10,11], leading to gap around the Fermi surface. Effects of impurity in holographic system were studied in [12] where they found phase transition of fermionic system from non Fermi liquid to Fermi liquid regime.

In addition to this flexible and informative approach, often it is advantageous to adopt the top-down approach where the dual theory is known and variations of parameters within the theory helps to make identification of states in the dual theory. Such an approach was employed to study probe branes and $N = 2$ supergravity theories [13–17]. Though Fermi surface was not found in the case of $N = 2$ supergravity theories [15–17], analyses of maximally symmetric gauged supergravity theories at zero temperature appear subsequently, leading to holographic Fermi surfaces [18–21]. Green's functions of dual operators at finite temperature were also computed for these theories [22,23]. A similar study for gravity background having vanishing entropy at zero temperature [24] reported fermionic fluctuations are stable within a gap around Fermi surface. Related discussions of Fermi surfaces appeared in [25–27].

There is an interesting class of backgrounds on the gravity side, which corresponds to condensation of charged scalar in holographic superconductors and gives rise to spontaneous breaking of $U(1)$ symmetry. Domain wall solutions are natural candidates for zero temperature limit of these backgrounds [28,29]. Study of fermions for such a condensed phase of holographic superconductor at zero temperature [30] shows a spectrum similar to that obtained in APRES experiment. Perhaps it is interesting to understand the mechanism lying behind the appearance of such a gapped spectrum in these cases. [31] considered a study of Majorana fermions with self coupling, coupled to a scalar (with twice charge) and found similar gapped spectrum. Holographic superconductors were constructed from string and M theory [13,32–35] and studies of spectra for these backgrounds appread in literature subsequently. [36] considered generic fermions with a domain wall solution as the background in a four dimensional supergravity, that follows from compactification of M theory. They obtained bands of normalisable modes in the region $\omega^2 < k^2$. Further studies of domain wall solutions with a symmetry breaking source in four dimensional gauged supergravity (dual to Aharony–Bergman–Jafferis–Maldacena (ABJM) theory) [37] shows both gapped and gapless bands of stable excitations. Studies of similar solutions dual to states in ABJM theory with broken $U(1)$ symmetry [38] also reported gapped spectrum. The gaps in the spectra have been attributed to low fermionic charge and particle-hole interaction [38].

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https://doi.org/10.1016/j.physletb.2018.03.037

The top-down approach has the advantage that it considers supergravity theories which are low energy limit of string/M theory for which, dual theory is known and holographic dictionary provides identification of operators in the dual field theory with various supergravity modes. Therefore, it provides an arena where one can study role of the dual operators underlying various phenomena and can address the field theory mechanism as well. In this vein, we consider a maximally symmetric gauged supergravity theory in seven dimensions, whose dual theory is a superconformal field theory in six dimensions and the operators dual to the various supergravity modes can be identified [39,40]. Being one of the three maximally superconformal field theories, it is interesting in its own right. Field contents involve a tensor multiplet and symplectic Majorana–Weyl gauginos which are different from theories in three and four dimensions that have been analysed in this context [37,38]. We have obtained a domain wall solution, interpolating between two AdS geometries, in this seven dimensional supergravity. The solution breaks a $U(1)$ of the R-symmetry group spontaneously and thus may correspond to zero temperature of holographic superconductor. We have computed the optical conductivity numerically and find that in the low frequency limit it behaves as $\omega^{2\triangle_B+3}$ for certain constant $\triangle_B$, while for high frequency it goes as ω^3 as expected for ultraviolet AdS_7 geometry. The fermionic content of the supergravity theory consists of 16 spin-1/2 fermions and we have considered only those modes, which do not couple to gravitino. In the background of the domain wall solutions we find there are only four such modes. We have studied spectral function for the operators dual to those modes and find in the spectrum there is a depleted region around $\omega = 0$. We have also artificially dialled the value of the charges up to $q = -2$ to study its effect on the spectrum and find the gap persists. Analysing the dual field theory, we find fermionic operators involving scalars with non zero expectation value, gives rise to gapped spectrum in these cases.

The plan of the article is as follows. In the next section, we describe the domain wall solution and study the optical conductivity. In section 3 we present Green's function while section 4 consists of the numerical result. We conclude with a discussion in section 5.

2. Domain wall solution

We begin with a discussion of bosonic content of the seven dimensional supergravity theory [41–43]. There exists a consistent truncation of eleven dimensional supergravity to this theory with only the lowest massless modes [41] and solution of this theory are expected to remain solution of the full theory, when uplifted [44]. It involves a gauged $SO(5)_g$ and a composite $SO(5)_c$ and consists of a graviton, a Yang–Mills gauge field in adjoint of $SO(5)_g$ and five rank-3 tensors in **5** of $SO(5)_g$. In addition, there are 14 scalar fields, which parameterise $SL(5,R)/SO(5)_c$. The lagrangian is given by,

$$
\begin{aligned}
2\kappa^2 e^{-1}\mathcal{L}_{boson} = {} & R + \frac{1}{2}m^2(T^2 - 2T_{ij}T^{ij}) - tr(P_\mu P^\mu) \\
& - \frac{1}{2}(V_I^i V_J^j F_{\mu\nu}^{IJ})^2 + m^2(V_i^{-1I} C_{\mu\nu\rho I})^2 \\
& + e^{-1}\Bigg(\frac{1}{2}\delta^{IJ}(C_3)_I \wedge (dC_3)_J \\
& + m\epsilon_{IJKLM}(C_3)_I F_2^{JK} F_2^{LM} + m^{-1} p_2(A,F)\Bigg)
\end{aligned}
\tag{2.1}
$$

Here $I, J = 1, 2, .., 5$ denote $SO(5)_g$ indices, and $i, j = 1, 2, .., 5$ denote $SO(5)_c$ indices. V_I^i represent fourteen scalar degrees of freedom parametrising $SL(5,R)/SO(5)_c$ coset transforming as **5** under both $SO(5)_g$ and $SO(5)_c$. The tensor T_{ij} is given by $T_{ij} = V_i^{-1\ I} V_j^{-1\ J}\delta_{IJ}$ and $T = T_{ij}\delta^{ij}$. There is a Yang–Mills gauge fields $A_{\mu I}^J$ transforming under adjoint of gauge group $SO(5)_g$ and covariant derivative of V_I^i is given as $\mathcal{D}_\mu V_I^i = \partial_\mu V_I^i - ig(A_\mu)_I^J\ V_J^i$. P_μ and Q_μ are symmetric and antisymmetric parts of the covariant derivative: $V_i^{-1I}\mathcal{D}_\mu V_I^k \delta_{kj} = (Q_\mu)_{[ij]} + (P_\mu)_{(ij)}$. In what follows we will set $C_{3I} = 0$.

The bosonic part of the full theory is quite involved, so we consider a truncation to make it simpler. Since the gauge group $SO(5)$ has rank two we have kept two Cartan gauge fields $A_\mu^{12} = A_\mu^{(1)}$ and $A_\mu^{34} = A_\mu^{(2)}$, while set other components of the gauge fields to be equal to zero. Considering a diagonal scalar vielbein V_I^i will lead to $U(1)^2$ gauge symmetry. Since we are interested in a background that would break one of the $U(1)$, we consider the following ansatz for the scalar vielbein

$$
\begin{aligned}
V_I^i &= \exp[\phi_2 Y_2]\exp[\phi_1 Y_1 + \phi_3 Y_3], \qquad Y_1 = \mathrm{diag}(1,-1,0,0,0), \\
Y_2 &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \oplus \mathrm{diag}(0,0,0), \qquad Y_3 = \mathrm{diag}(0,0,1,1,-2),
\end{aligned}
\tag{2.2}
$$

where Y_1, Y_2 and Y_3 are generators of $SL(5)$. For such a choice the bosonic action turns out to be

$$
\begin{aligned}
2\kappa^2 e^{-1}\mathcal{L} = {} & R - \frac{m^2}{2}V(\phi_1,\phi_3) - 2(\partial\phi_1)^2 \\
& - 2\sinh^2 2\phi_1(\partial_\mu\phi_2 + gA_\mu^{(1)})^2 - 6(\partial_\mu\phi_3)^2 \\
& - (F_{\mu\nu}^{(1)})^2 - e^{4\phi_3}(F_{\mu\nu}^{(2)})^2,
\end{aligned}
\tag{2.3}
$$

$$
\begin{aligned}
\text{where}\quad V(\phi_1,\phi_3) = {} & -(e^{2\phi_1} + e^{-2\phi_1} + 2e^{-2\phi_1} + e^{4\phi_3})^2 \\
& + 2(e^{4\phi_1} + e^{-4\phi_1} + 2e^{-4\phi_3} + e^{8\phi_3}).
\end{aligned}
$$

In the above lagrangian we will set $\phi_2 = 0$ that breaks the $U(1)$ symmetry associated with $A_\mu^{(1)}$. From (2.2) we can observe that this is equivalent to choice of unitary gauge for the coset. From the equations ensuing from the lagrangian (2.3) we can further simplify the action by setting $\phi_3 = 0$ and $A_\mu^{(2)} = 0$. From the equations of motion, that follows from the above lagrangian one can see that these correspond to consistent solution. The potential $V(\phi_1,\phi_3)$ will reduce to

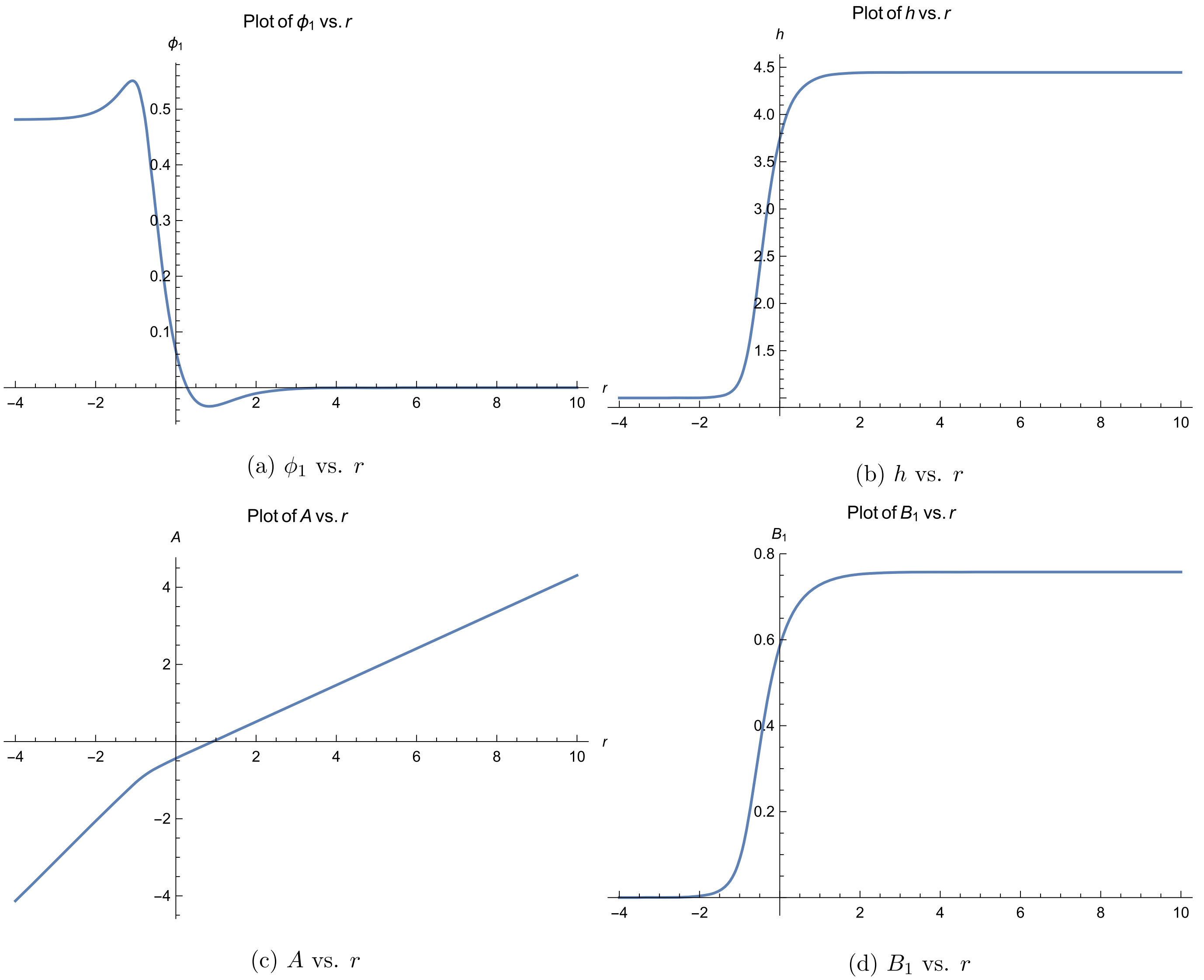
$$
V(\phi_1) = (2\cosh 2\phi_1 - 3)^2 - 16. \tag{2.4}
$$

The potential (2.4) has extrema at $\phi_1 = 0$ and $\phi_1 = \frac{1}{2}Log(\frac{3\pm\sqrt{5}}{2})$. We will consider a domain wall solutions such that the scalar ϕ_1 interpolates between these two extrema. In order to find such a solution, we choose the following ansatz for the metric and the gauge field

$$
ds^2 = e^{2A}(-hdt^2 + d\vec{x}_5^2) + \frac{dr^2}{h}, \qquad A^{(1)} = B_1 dt. \tag{2.5}
$$

With this ansatz, the equations of motion turn out to be

$$
\begin{aligned}
-5A'' &= 2\frac{e^{-2A}}{h^2}g^2\sinh^2 2\phi_1 B_1^2 + 2\phi_1'^2, \\
h'' + 6A'h' &= 4\frac{e^{-2A}}{h}g^2\sinh^2 2\phi_1 B_1^2 + 4e^{-2A}B_1'^2, \\
4h[\phi_1'' + (6A' + \frac{h'}{h})\phi_1'] &= -4\frac{e^{-2A}}{h}\sinh 4\phi_1 g^2 B_1^2 + \frac{m^2}{2}\frac{\partial V}{\partial\phi_1},
\end{aligned}
$$



(a) ϕ_1 vs. r (b) h vs. r (c) A vs. r (d) B_1 vs. r

Fig. 1. Plots of different fields for domain wall solution.

$$B_1'' + 4A'B_1' = \frac{1}{h} g^2 \sinh^2 2\phi_1 B_1,$$
$$30(A')^2 + 5\frac{h'}{h}A' = 2\frac{e^{-2A}}{h^2} g^2 \sinh^2 2\phi_1 B_1^2 + 2\phi_1'^2$$
$$-2\frac{e^{-2A}}{h} B_1'^2 - \frac{m^2}{2}\frac{1}{h} V(\phi_1), \tag{2.6}$$

where the last equation is a constraint. Once other equations are satisfied, this will remain valid over the range of r provided it is satisfied at some value of r.

We look for solution of above equations (2.6) with $\phi_1 = \phi_{IR} = \frac{1}{2} Log(\frac{3\pm\sqrt{5}}{2})$ at IR ($r \to -\infty$) and $\phi_1 = 0$ at UV ($r \to \infty$). The IR limit of the equation admits the following solution

$$\phi_1 \sim \phi_{IR}, \quad h = 1, \quad B_1 = 0, \quad A = \frac{r}{L_{IR}}, \tag{2.7}$$

with exponentially suppressed corrections. With this solution, IR geometry turns out to be AdS with a radius $L_{IR} = \frac{\sqrt{15}}{2m}$. Similarly at the UV limit, the geometry is also AdS with radius $L_{UV} = \frac{2}{m}$ with $A = \frac{r}{L_{UV}}$.

In order to obtain a solution interpolating between these two extremes we need to specify the first corrections to the scalar and the gauge fields given in (2.7) at IR. We choose the following corrections [32,33,45]

$$\phi_1(r) = \phi_{IR} + a_\phi e^{\triangle_\phi (r/L_{IR})}, \qquad B_1(r) = a_B e^{\triangle_B (r/L_{IR})}, \tag{2.8}$$

where $\triangle_\phi = \frac{\sqrt{111}}{2} - 3$ and $\triangle_B = \frac{\sqrt{91}}{2} - 2$, as obtained from the equations of ϕ and B at the IR limit. As explained in [33] a_b can be set to 1 by shifting r and rescaling t and x. Therefore, only parameter for the solution remains to be a_ϕ.

At the UV limit the equation of motion implies ϕ_1 behaves as e^{-2A} and e^{-4A}, which correspond to the source and expectation value of the dual operator respectively. Since we are interested in the solution, that will break the symmetry spontaneously, we impose the additional restriction that $\lim_{r\to\infty} \phi_1 \sim e^{-4A}$. With this restriction, the parameter a_ϕ can have only discrete values.

We have solved the equations (2.6) numerically subject to the boundary condition (2.8). We choose the value of a_ϕ to be 1.717 as that gives the scalar field with least number of nodes and so corresponds to a solution most likely to be stable. The solutions of the different fields are given in Fig. 1. The relative speed of propagation of light in the ultraviolet and the infrared, given by the index

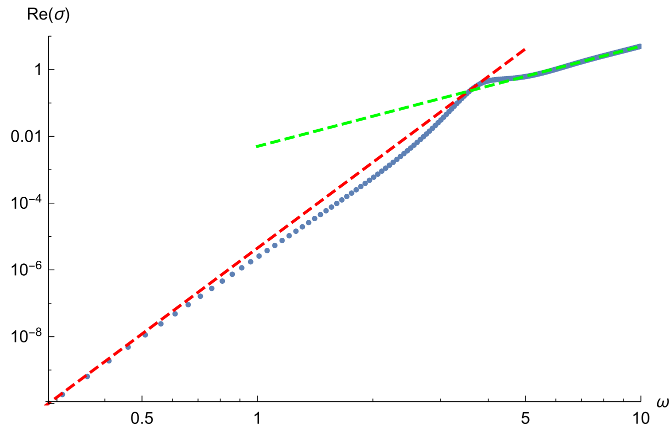


Fig. 2. Re(σ) vs. ω.

of refraction, $n = \sqrt{h_{UV}/h_{IR}}$ [32,33,45] turns out to be 2.10845 in this case.

After obtaining the domain wall solution we study behaviour of conductivity with variation of frequency [32,33,45,46]. We add a time dependent perturbation to the gauge field in x direction, given by $A_x = a_x(r)e^{-i\omega t}$. Turning on a gauge field will also produce a metric perturbation in g_{tx}. From the Maxwell's equation and Einstein equation one will get two coupled linearised equation involving the perturbations, a_x and δg_{tx}, which give rise to the following equation for a_x,

$$a_x'' + (4A' + \frac{h'}{h})a_x' + \frac{\omega^2}{h^2}e^{-2A}a_x - \frac{4}{h}e^{-2A}(B_1')^2 a_x = \frac{g^2}{h}\sinh^2 2\phi_1 a_x. \quad (2.9)$$

The asymptotic behaviour of a_x is given by

$$a_x(r) \sim a_x^{(0)} + a_x^{(4)}e^{-4A} + ..., \quad (2.10)$$

where the ellipses denote higher order terms. Solving the equation (2.9) subject to the infalling boundary condition one can obtain the expression of conductivity as

$$\sigma \sim \frac{-i}{\omega}\frac{a_x^{(4)}}{a_x^{(0)}}, \quad (2.11)$$

where the constant of proportionality does not depend on ω.

At IR the infalling solution is given by

$$a_x(r) = e^{-2r/L_{IR}}H^{(1)}_{\triangle_B+2}(\omega L_{IR}e^{-r/L_{IR}}), \quad (2.12)$$

where $H^{(1)}$ is Hankel function of first kind.

We have numerically solved (2.9) subject to the boundary condition (2.12) and evaluate conductivity as given in (2.11). We have plotted conductivity vs. frequency in the Fig. 2. In order to find out the behaviour of conductivity for small ω we introduce $\mathcal{F}$ [33,45] as

$$\mathcal{F} = -he^{4A}\frac{a_x^*\partial_r a_x - a_x\partial_r a_x^*}{2i}, \quad (2.13)$$

where $\partial_r\mathcal{F} = 0$, as follows from (2.9). At the IR limit, from (2.12) one can see that $\mathcal{F}$ is independent of ω. Real part of σ is given by

$$Re(\sigma) = \frac{\mathcal{F}}{4h_{UV}}\frac{1}{\omega|a_x^{(0)}|^2}, \quad (2.14)$$

and so in order to determine ω dependence we need to find out how $a_x^{(0)}$ depends on ω. For the region $L_{IR}Log(\omega L_{IR}) << r << r_{IR}$ where deviation of geometry from AdS$_{IR}$ is negligible one can show,

$$a_x \sim -i\frac{\Gamma(\triangle_B+2)}{\pi}(\frac{2}{\omega L_{IR}})^{\triangle_B+2}Z_x(r), \quad (2.15)$$

where at the IR region, $\lim_{r\to-\infty} e^{-\triangle_B r/L_{IR}}Z_x(r) \to 1$. For large r the ω^2 term in (2.9) is negligible and on the basis of that if we assume following [33,45] similar ω dependence of a_x will be valid for large r as well, then $a_x^{(0)} = \lim_{r\to\infty} a_x(r) \sim \omega^{-(\triangle_B+2)}$. From (2.14) it follows

$$Re(\sigma) \sim \omega^{2\triangle_B+3}. \quad (2.16)$$

From Fig. 2 one can observe that ω dependence of $Re(\sigma)$ agrees with this for small ω. For large ω limit, real part of conductivity goes as ω^3, which is its behaviour for ultraviolet AdS$_7$ geometry.

3. Fermionic action

The $N = 4$ gauged supergravity in seven dimensions [41–43] consists of two kinds of fermions. One is gravitino ψ_μ^A with spin-3/2 transforming under $SO(5)_c$ as **4**. Other is spin-1/2 field λ_i^A which transform as **16** under $SO(5)_c$ and satisfy $\gamma^i\lambda_i = 0$. A ($A = 1,..,4$) and i ($i = 1,..,5$) are spinor and vector indices respectively of $SO(5)_c$. The terms in the Lagrangian consisting of only spin-1/2 fields λ^i are given by [42]

$$e^{-1}\mathcal{L}_{fermion} = -\frac{1}{2}\bar{\lambda}_i(\Gamma^\mu D_\mu\lambda^i) - \frac{m}{8}\bar{\lambda}_i(8T^{ij} - T\delta^{ij})\lambda_j + \frac{1}{32}\bar{\lambda}_i\gamma^j\gamma^{kl}\gamma^i\Gamma^{\mu\nu}\lambda_j(F_{\mu\nu})_{kl}, \quad (3.1)$$

representing kinetic term, mass term and Pauli term. The covariant derivatives for our background are given by

$$D_\mu\lambda^i = \nabla_\mu\lambda^i - ig\cosh 2\phi_1[A_\mu^{(1)}(J^{12})^i_j)\lambda^j + (A_\mu^{(1)}S^{12})\lambda^i], \quad (3.2)$$

where J^{12} and S^{12} are the vector and spinor representations of the generators of the gauge group $U(1)$. ∇_μ is the covariant derivative containing the spin connection and is given by

$$\nabla_\mu = \partial_\mu - \frac{1}{4}(\omega_\mu)_{ab}\Gamma^{ab}. \quad (3.3)$$

The terms in the Lagrangian corresponds to coupling between gravitino ψ_μ and spin-1/2 fields λ^i are given by

$$e^{-1}\mathcal{L}_{int} = \bar{\psi}_\mu(-m\Gamma^\mu T_{ij}\gamma^i\lambda^j + \Gamma^\nu\Gamma^\mu(P_\nu)_{ij}\gamma^i\lambda^j + \frac{1}{2}\Gamma^{\nu\sigma}\Gamma^\mu(F_{\nu\sigma})_{ij}\gamma^i\lambda^j). \quad (3.4)$$

In order to simplify the notation we will use for $SO(5)$ vector indices: $v^{1\pm} = \frac{1}{\sqrt{2}}(v^1 \pm iv^2)$, $v^{2\pm} = \frac{1}{\sqrt{2}}(v^3 \pm iv^4)$, $v^0 = v^5$, so that $U(1)\times U(1)$ charges associated are as follows: $v^{1\pm}$ and $v^{2\pm}$ have charges $(\mp1, 0)$ and $(0, \mp1)$ respectively while v^0 has charge $(0, 0)$. For $SO(5)$ spinor indices we use $\lambda(s_{12}, s_{34})$, where $s_{12}, s_{34} = \pm\frac{1}{2}$ are eigenvalues of $S^{12} = -(i/2)\gamma^1\gamma^2$ and $S^{34} = -(i/2)\gamma^3\gamma^4$ respectively. With this notation, 16 independent components of λ can be organised as $\lambda^{1\pm}(s_{12}, s_{34})$ and $\lambda^{2\pm}(s_{12}, s_{34})$.

We would like to study the behaviour of fermions that do not couple to gravitino. The second $U(1)$ (associated with $A_\mu^{(2)}$) remains unbroken and gravitino has charges $\pm\frac{1}{2}$ with respect to it. Therefore spinor components with total charge $\pm\frac{3}{2}$ with respect to second $U(1)$ will not couple to gravitino. So $\lambda^{2-}(s_{12}, \frac{1}{2})$ and $\lambda^{2+}(s_{12}, -\frac{1}{2})$, with $s_{12} = \pm\frac{1}{2}$ are decoupled from gravitino and we restrict ourselves to the case of these four fermions only. An explicit computation shows, for the present choice of V_I^i, all the other fermions in **16** couple to gravitino.

Dirac equation satisfied by these fermions λ_i can be written as

$$(\Gamma^\mu D_\mu + \frac{m}{4}(5 - 2\cosh 2\phi_1) + i\frac{s_{12}}{2}\Gamma^{\mu\nu}F^{(1)}_{\mu\nu})\lambda = 0,$$
$$\text{where,} \quad D_\mu\lambda = \partial_\mu\lambda - ig(q\cosh 2\phi_1 A^{(1)}_\mu)\lambda, \quad F^{(1)}_{\mu\nu} = \partial_\mu A^{(1)}_\nu - \partial_\nu A^{(1)}_\mu, \tag{3.5}$$

q are the charges associated with gauge field and is given by s_{12}, but we have kept it generic. Both the mass term and the charge term depend on the scalar field ϕ_1. At the IR limit $\phi_1 = \phi_{IR}$ and the mass turns out to be $m/2$, while at the UV limit $\phi_1 = 0$ and the mass becomes $3m/4$.

We have chosen the following seven dimensional Γ-matrices:

$$\Gamma^{\hat{t}} = \begin{pmatrix} \Gamma_1^{\hat{t}} & 0 \\ 0 & \Gamma_1^{\hat{t}} \end{pmatrix}, \quad \Gamma^{\hat{r}} = \begin{pmatrix} \Gamma_1^{\hat{r}} & 0 \\ 0 & \Gamma_1^{\hat{r}} \end{pmatrix}, \quad \Gamma^{\hat{i}} = \begin{pmatrix} \Gamma_1^{\hat{i}} & 0 \\ 0 & \Gamma_1^{\hat{i}} \end{pmatrix}$$
$$\Gamma_1^{\hat{t}} = \begin{pmatrix} 0 & i\sigma_2 \\ i\sigma_2 & 0 \end{pmatrix}, \quad \Gamma_1^{\hat{r}} = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \Gamma_1^{\hat{i}} = \begin{pmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{pmatrix}, \tag{3.6}$$

where I is 2×2 is identity matrix, σ_1, σ_2 and σ_3 are Pauli spin matrices. Due to the fact that relevant Γ-matrices have identical copies in both the diagonal blocks, we can choose, $\lambda = (\psi, \chi)^T$ where both the 4-component spinors ψ and χ will satisfy the same equations, and it is sufficient to consider only the upper component ψ. We absorb the effect of spin connection by suitably redefining the spinors with appropriate prefactor and choose to write $\psi = (\psi^+, \psi^-)^T$, and each component satisfies the following equation:

$$(\sqrt{h}\partial_r + \frac{m}{4}(5 - 2\cosh 2\phi_1))\psi^+ + ie^{-A}[k\sigma_1 - (\frac{\omega + gq\cosh 2\phi_1 B_1}{\sqrt{h}} - s_{12}B_1')i\sigma_2]\psi^- = 0,$$
$$(-\sqrt{h}\partial_r + \frac{m}{4}(5 - 2\cosh 2\phi_1))\psi^- + ie^{-A}[k\sigma_1 - (\frac{\omega + gq\cosh 2\phi_1 B_1}{\sqrt{h}} + s_{12}B_1')i\sigma_2]\psi^+ = 0. \tag{3.7}$$

We further write the two component spinors as $\psi^\pm = (\psi_1^\pm, \psi_2^\pm)$ and obtain the equations for individual components,

$$(\sqrt{h}\partial_r - \frac{m}{4}(5 - 2\cosh 2\phi_1))\psi_1^- - ie^{-A}[k - (\frac{\omega + gq\cosh 2\phi_1 B_1}{\sqrt{h}} + s_{12}B_1')]\psi_2^+ = 0,$$
$$(\sqrt{h}\partial_r + \frac{m}{4}(5 - 2\cosh 2\phi_1))\psi_2^+ + ie^{-A}[k + (\frac{\omega + gq\cosh 2\phi_1 B_1}{\sqrt{h}} - s_{12}B_1')]\psi_1^- = 0. \tag{3.8}$$

As one can observe, the Dirac equation reduces to coupled equations for the two components (ψ_1^-, ψ_2^+). A similar set of equations follows for the other two components,

$$(\sqrt{h}\partial_r - \frac{m}{4}(5 - 2\cosh 2\phi_1))\psi_2^- - ie^{-A}[k + (\frac{\omega + gq\cosh 2\phi_1 B_1}{\sqrt{h}} + s_{12}B_1')]\psi_1^+ = 0,$$
$$(\sqrt{h}\partial_r + \frac{m}{4}(5 - 2\cosh 2\phi_1))\psi_1^+ + ie^{-A}[k - (\frac{\omega + gq\cosh 2\phi_1 B_1}{\sqrt{h}} - s_{12}B_1')]\psi_2^- = 0. \tag{3.9}$$

These two sets of equations are related through flipping of signs of ω, q and s_{12}. In what follows, we will be considering only the first set of equations (3.8). These equations cannot be solved analytically and we will use numerical computation to solve these using appropriate boundary conditions. However, at the IR and UV limits one can find analytic expressions for the behaviour of the solutions.

IR limit: At the IR limit we have $A \sim r/L_{IR}$, $h \sim 1$, $B_t^{(1)} \sim 0$, $\phi_1 \sim \phi_{IR}$. The geometry is AdS with radius L_{IR} and the mass reduces to $m_{IR} = m/2$. In order to solve the equations (3.8) we choose the infalling boundary condition [47], which are given by as follows: For space-like momenta, $k^2 \geq \omega^2$:

$$\psi_1^-(r) = e^{-r/2L_{IR}} K_{m_{IR}L_{IR}+\frac{1}{2}}(\sqrt{k^2 - \omega^2}L_{IR}e^{-r/L_{IR}}),$$
$$\psi_2^+(r) = i\sqrt{\frac{k+\omega}{k-\omega}}e^{-r/2L_{IR}} K_{-m_{IR}L_{IR}+\frac{1}{2}}(\sqrt{k^2 - \omega^2}L_{IR}e^{-r/L_{IR}}), \tag{3.10}$$

where $K_{\pm m_{IR}L_{IR}+\frac{1}{2}}$ are modified Bessel function. For time-like momentum, $\omega > |k|$ they are expressed in terms of Hankel function of first kind,

$$\psi_1^-(r) = e^{-r/2L_{IR}} H^{(1)}_{m_{IR}L_{IR}+\frac{1}{2}}(\sqrt{\omega^2 - k^2}L_{IR}e^{-r/L_{IR}}),$$
$$\psi_2^+(r) = -i\sqrt{\frac{\omega+k}{\omega-k}}e^{-r/2L_{IR}} H^{(1)}_{m_{IR}L_{IR}-\frac{1}{2}}(\sqrt{\omega^2 - k^2}L_{IR}e^{-r/L_{IR}}). \tag{3.11}$$

Similarly, for $\omega < -|k|$ they are expressed in terms of Hankel function of second kind.

UV limit: At the UV limit, we have $A \sim r/L_{UV}$, $h \sim h_{UV}$, $\phi_1 \sim 0$. The geometry is once again AdS with radius $L_{UV} = 2/m$ and the mass reduces to $m_{UV} = 3m/4$. In this limit the equations of fermions depend only on the mass terms and the asymptotic limit of the solutions are given by,

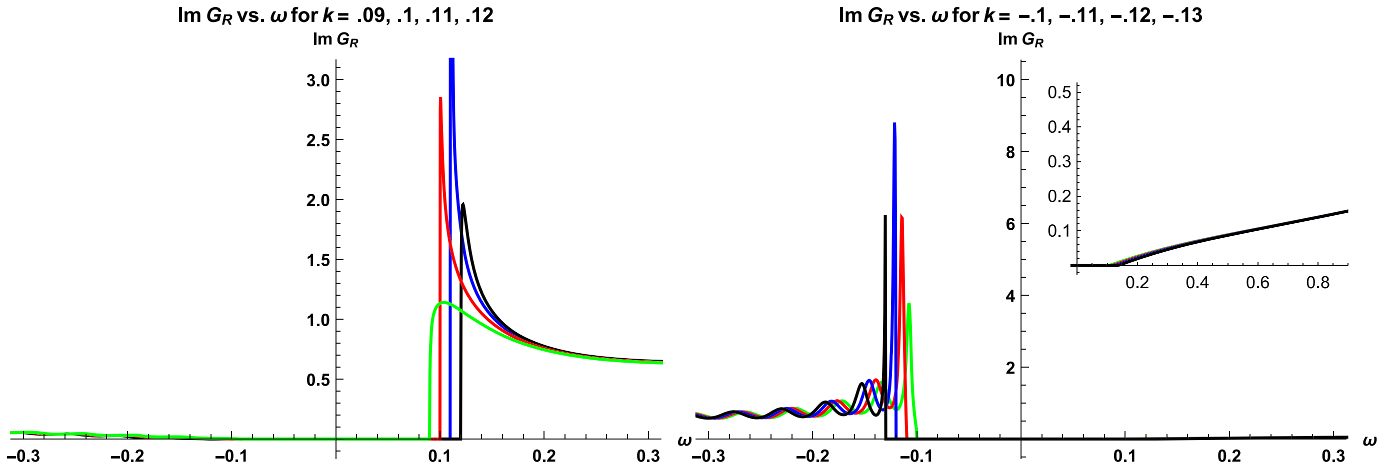
$$\psi_1^-(r) \sim C_1^- e^{m_0 r} + D_1^- e^{-(m_0+1)r},$$
$$\psi_2^+(r) \sim C_2^+ e^{(m_0-1)r} + D_2^+ e^{-m_0 r}, \tag{3.12}$$

where we have set $m = 2$ and defined $m_0 = m_{UV}/\sqrt{h_{UV}}$.

For this set of fermions (ψ_1^-, ψ_2^+) Green function is given by the following [6].

$$G_R(\omega, k) = i\frac{D_2^+}{C_1^-}, \tag{3.13}$$

while the other set (ψ_1^+, ψ_2^-) leads to a similar expression. These two comprises two diagonal components of Green's function. In what follows, we will consider only the one given in (3.13). The spectral function is given by $A(\omega, k) = Im[G_R(\omega, k)]$. We are interested in the spectrum as function of frequency and momenta. In the next section, we will numerically solve the Dirac equations for boundary conditions at IR.



(a) ImG_R vs. ω for $q = 1/2$

(b) ImG_R vs. ω for $q = -1/2$

Fig. 3. Spectral function for fermionic mode with: Left: $k = .09$ (green), $k = .1$ (red), $k = .11$ (blue), $k = .12$ (black). Right: $k = -.1$ (green), $k = -.11$ (red), $k = -.12$ (blue), $k = .13$ (black). Coloured figure(s) are available in the web version of this article.

We conclude this section with the discussion of the dual field theory. According to the conjecture [1–3] the dual field theory is given by six dimensional $(2, 0)$ conformal field theory. It has an R-symmetry group $SO(5)$. Relevant field content is tensor multiplet consisting of a self-dual 2-form potential $B_{\mu\nu}$ transforming as **1**, five scalars Σ^i transforming as **5** and four symplectic Majorana–Weyl spinors ψ^A transforming as **4** under the R-symmetry group. In our notation, $U(1) \times U(1)$ charges associated with various fields are as follows: $\Sigma^{1\pm}$ and $\Sigma^{2\pm}$ have charges $(\mp 1, 0)$ and $(0, \mp 1)$ respectively while charge of Σ^0 is $(0, 0)$. Four spinors can be represented as $\psi(s_{12}, s_{34})$ with $s_{12}, s_{34} = \pm 1/2$.

In order to obtain the operators dual to the supergravity fields, B, Σ and ψ are taken to be in the adjoint representation of $U(N)$ [39,40]. Operators dual to the spinors in the supergravity transforming under **16** are of the form $tr(\Sigma\psi)$. So counting the $U(1) \times U(1)$ charges, those operators may be organised as $tr(\Sigma^{1\pm}\psi(s_{12}, s_{34}))$ and $tr(\Sigma^{2\pm}\psi(s_{12}, s_{34}))$ with $s_{12}, s_{34} = \pm 1/2$.

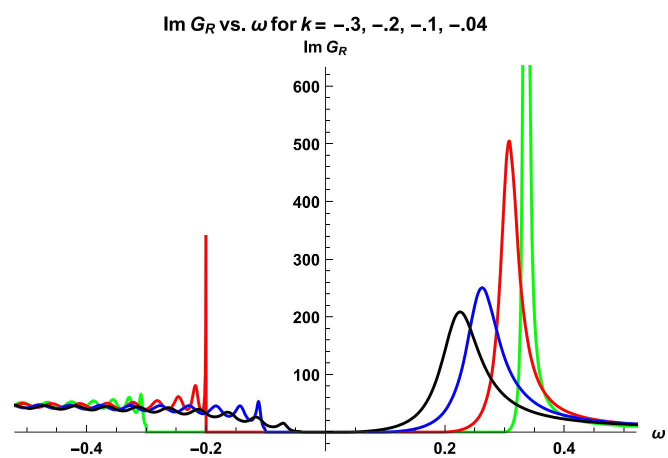
The operators dual to the 14 scalars appearing in the coset are identified in the following way [48]. The symmetric tensor, T^{ab}, $a, b = 1, 2.., 5$ can be written as $(e^S)_{ab}$, where S_{ab} is a 5×5 traceless matrix. The operators dual to S_{ab} are given by $\mathcal{O}_{ab} = \Sigma_a \Sigma_b - \frac{1}{5}\delta_{ab}(\Sigma_c \Sigma^c)$. For our choice of scalars,

$$\begin{aligned} S_{ab} &= (-2\phi_1, 2\phi_1, 2\phi_3, 2\phi_3, -4\phi_3), \\ \mathcal{O}_{\phi_1} &\sim -(\Sigma_1^2 - \frac{1}{5}(\Sigma_1^2 + \ldots + \Sigma_5^2)) \quad \text{etc.} \end{aligned} \tag{3.14}$$

In this context it may be observed that there is a $U(1)$ symmetry that rotates between Σ_3 and Σ_4 and the fermionic operators dual to the four fermionic modes in the supergravity that we are considering involve Σ_3 and Σ_4 only.

4. Numerical result

In this section we have plotted the spectral function corresponding to the operator in the boundary CFT dual to the fermionic mode $\lambda^{2\pm}(\frac{1}{2}, \pm\frac{1}{2})$. The charges with respect to the first $U(1)$, $q = \pm\frac{1}{2}$ and the coefficient of the Pauli term is $\pm\frac{1}{2}$. The plots are given in Fig. 3 for different values of k. For positive q, given in Fig. 3a, one can observe, it shows a peak on the positive ω. The peak is highest for $k = 0.11$ and as k deviates from it the peak height decreases. On the negative side of ω it develops a hump.



ImG_R vs. ω for $q = 2$

Fig. 4. Spectral function for fermionic mode with $k = -.3$ (green), $k = -.2$ (red), $k = -.1$ (blue), $k = -.04$ (black). Coloured figure(s) are available in the web version of this article.

For negative value of q, given in Fig. 3b, the peaks appear on the negative side of ω and the highest peak appears at $k = -.12$. As k deviates from it peak becomes smaller. Humps appear once again, on the positive side of ω, this time. Since the humps are not visible in the main figure we have provided a magnified picture for positive ω in the inset. In both the cases, spectral function shows a depleted region around $\omega = 0$ indicating a gap in the spectrum. We have checked the spectral function for further higher values of $|q|$ upto $q = -2$. We find the depleted region continues to exist. However, more peaks appear as we increase the $|q|$ and heights of the peaks also increase.

In the present case, asymptotic charges of fermions under the U(1) gauge group and Pauli terms are given by $\pm\frac{1}{2}$ and $\pm\frac{1}{4}$ respectively. We have artificially dialled q to $q = -2$ and set Pauli term equals zero in order to check the effect of charge and Pauli term on the spectrum. Plot of the spectral function of the dual operator is given in Fig. 4. As we observe, the gap persists. With higher $|q|$ peaks starts appearing on the positive side of ω as well and these peaks are much higher and sharper compared to $q = \pm 1/2$. On the negative side, humps continue to exist along with small peaks for this value of q.

5. Discussion

We have obtained a domain wall solution in seven dimensional gauged supergravity, which interpolates between two AdS geometries of different radii. The scalar field takes a non-zero value at the IR, while at the UV it vanishes, both of which correspond to extrema of the potential. Presence of the scalar field breaks one of the $U(1)$ in the Cartan of $SO(5)$ spontaneously, and in that sense it is similar to holographic superconductor. We have studied the optical conductivity and find for small frequency real part of conductivity obeys a power law $\omega^{2\triangle_B+3}$ for certain constant $\triangle_B$, while for large frequency it goes as cubic power of frequency as expected for ultraviolet AdS_7 geometry.

We have considered the fermions with the domain wall as the background. It turns out, there are four fermionic modes (out of total 16) in the supergravity theory, that do not couple to the gravitino in this background. We have studied the spectral function associated with the dual operators to those modes and find all of them leads to gapped spectra. We have artificially increased the charge to $|q| = 2$ and set the Pauli term equals zero, and find the gap remains in the spectral function. Similar gapped spectrum was obtained in [30] for fermionic quasi particles in presence of condensate at zero temperature. Gapped spectra were also found in four dimensional gauged supergravity dual to ABJM model [37, 38], where the gap has been attributed to the low charge or particle hole interaction.

In order to analyse the field theory aspects, we consider the scalars in the dual field theory. Since the domain wall solution corresponds to expectation value of the scalars, they may play a role determining the spectrum. Indeed, as shown in [19] for $\mathcal{N} = 4$ SYM theory, scalars play important role in determining nature of the Fermi surface. From the asymptotic behaviour of ϕ_1 in the UV limit given in Fig. 2, we observe it corresponds to expectation value of dual operator $\mathcal{O}_{\phi_1}$ to be negative. Furthermore, since $\phi_3 = 0$ for our solution, we can assume $\mathcal{O}_{\phi_3} = 0$. From the relation (3.14), it turns out $\langle tr(\Sigma_3^2) + tr(\Sigma_4^2)\rangle > 0$. So either Σ_3 or Σ_4 or both have non-zero expectation value. On the other hand, the fermionic modes we have considered are dual to $tr(\Sigma^{2\pm}\psi(s_{12}, s_{34}))$ with $s_{12}, s_{34} = \pm 1/2$ and $\Sigma^{2\pm} = \frac{1}{\sqrt{2}}(\Sigma^3 \pm i\Sigma^4)$. Therefore in the present case, the fermionic operators involving bosons with non zero expectation value, gives rise to gapped spectrum. It will be interesting to understand the essential field theory mechanism that determines appearance of gap in the spectrum.

The domain wall we have considered interpolates between two conformal field theories and in that sense it is similar to renormalisation group flow. However, in this case it has a spontaneously generated expectation value of a symmetry breaking operator rather than adding relevant deformation. It may be interesting to address stability issues associated with this domain wall solution. It would also be interesting to obtain this solution as zero temperature limit of holographic superconductor. Regarding the fermionic spectrum, a clear picture of the roles played by various operators in the dual field theory would be quite useful, which requires models that can provide more precise and detailed analysis. In order to keep the analysis simple we have set one of the gauge fields and scalar fields to be zero. Considering a more general background may be more informative. We have not considered the fermionic modes coupled to gravitini and a full fledged analysis of all the modes including gravitini may lead to further insight.

Acknowledgement

The work of NR is supported by University Grants Commission of India (UGC India).

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